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Ph.D. Entrance Examination, 2024

MATHEMATICS

Maximum Marks : 50

Note : Each question carrying 2 marks.

Q. 1. Find the supremum of the set :

$$S = \left\{ 1 + \frac{(-1)^n}{n} : n \in N \right\}$$

(a) 0

(b) $\frac{1}{2}$

(c) $\frac{3}{2}$

(d) –1

Q. 2. If $<s_n>$ is the sequence defined by $s_n = (-1)^n \left(1 + \frac{1}{n}\right)$, then lim inf S_n is :

(a) –1

(b) 1

(c) 0

(d) 2

Q. 3. The function f defined on R^+ as $f(x) = \sin \frac{1}{x}$, $\forall x > 0$ is :

(a) Continuous and uniformly continuous on R^+

(b) Uniformly continuous but not continuous on R^+

(c) Continuous but not uniformly continuous on R^+

(d) Neither continuous nor uniformly continuous on R^+

Q. 4. A function f is defined on [0, 1] by

$$f(x)=\begin{cases}\frac{1}{n} & \text{for} \quad \frac{1}{n+1}<x\le\frac{1}{n}, n=1,2,3,\ldots\ldots\ldots \\ 0 & \text{for} \quad x=0\end{cases}$$

then the value of $(R)\int_0^1 f(x)dx$ is :

(a) 0

(b) $\frac{\pi}{2}$

(c) $\frac{\pi^2}{6}$

(d) $\frac{\pi^2}{6}-1$

Q. 5. The integral $\int_0^\infty x^{n-1} e^{-x} dx$ is convergent for :

(a) n < 0

(b) n > 0

(c) n > 1

(d) $n \ge 1$

Q. 6. If V is the vector space of dimension 2 over $Z_2=\{\bar{0},\bar{1}\}$, then the number of basis is :

(a) 1

(b) 2

(c) 3

(d) 4

Q. 7. For what value of λ and μ the system of equations :

$x + y + z = 6$

$x + 2y + 3z = 10$

$x + 2y + \lambda z = \mu$

have no solution ?

(a) $\lambda \neq 3$ and $\mu = 10$

(b) $\lambda = 3$ and $\mu = 10$

(c) $\lambda = 3$ and $\mu \neq 10$

(d) $\lambda \neq 3$ and $\mu = 1$

Q. 8. If $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ is an eigen vector of $\begin{bmatrix} 1 & -n \\ -3 & 2n \end{bmatrix}$, then the value of n is :

(a) –2

(b) 2

(c) 1

(d) –1

Q. 9. For what value of z the function w defined by $z = e^{-v}(\cos u + i \sin u)$ where $w = u + iv$ ceases to be analytic ?

(a) $z = 0$

(b) $z = i$

(c) $z = -i$

(d) $z = \pm 1$

Q. 10. Find the radius of convergence of the power series $\sum \left(1 + \frac{1}{n}\right)^{n^2} z^n$:

(a) e

(b) $\frac{1}{e}$

(c) 1

(d) 4

Q. 11. If C is the circle $|z| = 1$, then the value of $\int_C \frac{e^z}{z-2} dz$ is :

(a) 0

(b) e^2

(c) e

(d) –1

Q. 12. Find the residue of $\dfrac{z^2 - 2z}{(z+1)^2(z^2+4)}$ at z = –1 :

(a) $\dfrac{7+i}{25}$

(b) $\dfrac{7-i}{25}$

(c) $\dfrac{14}{25}$

(d) $-\dfrac{14}{25}$

Q. 13. Find the bilinear transformation which maps the points z = 1, 0, –1 of z-plane into w = i, 0, –i of w-plane :

(a) w = 1 + iz

(b) w = iz

(c) $w = \dfrac{z+i}{-3z+i}$

(d) $w = i\left(\dfrac{2+z}{2-3z}\right)$

Q. 14. Find total number of subgroups of $(z_{12}, +)$

(a) 3

(b) 4

(c) 12

(d) 6

Q. 15. If G is a cylic group of order 8, then the number of isomorphisms from G to G is :

(a) 2

(b) 4

(c) 8

(d) 16

Q. 16. The number of non-abelian groups of order 6 is :

(a) 1

(b) 2

(c) 3

(d) 6

Q. 17. The degree of $\sqrt{2}+\sqrt{5}$ over Q is :

(a) 1

(b) 3

(c) 4

(d) 6

Q. 18. Which of the following is not true ?

(a) Every Euclidian ring is a principal ideal ring

(b) In a commutative ring without unity, a maximal ideal will always be prime

(c) A field has no proper ideals

(d) In a unique factorization domain D, an element is prime if it is irreducible

Q. 19. If the solution of the partial differential equation $\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y} = x + y + z$ is :

$\phi\left[x-y, e^{-\alpha x}(\beta+x+y+z)\right]=0,$ then the value of

α and β are :

(a) $\alpha = 1, \beta = 0$

(b) $\alpha = 1, \beta = 1$

(c) $\alpha = 0, \beta = 2$

(d) $\alpha = 1, \beta = 2$

Q. 20. The partial differential equation

$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}+2\frac{\partial^2 u}{\partial y \partial z}=0$ is :

(a) Hyperbolic

(b) Elliptic

(c) Parabolic

(d) Circular

Q. 21. If the solution of partial differential equation

$$(D - 1)(D - D' + 1)z = 1 + xy$$

is

$z=e^{x}\phi_1(y)+e^{-\alpha x}\phi(y+x)-x^{\beta}y-x,$ then the possible values of α, β are :

(a) $\alpha = 1, \beta = 1$

(b) $\alpha = -1, \beta = 1$

(c) $\alpha = 1, \beta = -1$

(d) $\alpha = 0, \beta = 1$

Q. 22. Which of the following is not true ?

(a) The product of two separable topological spaces is separable

(b) A subset of real line containing at least two points is connected if and only if it is an interval

(c) Connectedness is a hereditary property

(d) Every compact subset of a compact topological space is not necessarily closed

Q. 23. Consider the following statements :

(i) Every subspace of a first countable space is first countable

(ii) Every second countable topological space is separable

Which of the following options is/are correct ?

(a) Only (i) is true

(b) Only (ii) is true

(c) Neither (i) nor (ii) is true

(d) Both (i) and (ii) are true

Q. 24. Which of the following is true ?

(a) Every Hilbert space is reflexive

(b) An orthonormal set in a Hilbert space is linearly dependent

(c) If $\{e_i\}$ is an orthonormal set in a Hilbert space H, then for every vector x in H

$$\sum \left|(x, e_i)\right|^2 \geq \|x\|^2$$

(d) Every complete subspace of a normed linear space is not necessarily closed

Q. 25. If M is a closed linear subspace of a Hilbert space H, then $H = M \oplus M^{\perp}$. This theorem is called :

(a) Parseval identity

(b) Riesz lemma

(c) Riesz representation theorem

(d) Projection theorem